Sitting on the Dock of the Bayes

Applying Bayesian Methods to Modern Real Estate Appraisal

by Jim Amorin, MAI, SRA, AI-GRS

Abstract

This article explores the application of Bayes' Theorem in real estate appraisal, highlighting how statistical reasoning can enhance property valuation. Moving beyond traditional point estimates, Bayesian inference enables appraisers to quantify uncertainty, incorporate professional judgment, and refine valuations as new market evidence emerges. The approach transforms subjective adjustments into probability distributions, producing defensible confidence intervals rather than single-point values. Through illustrated examples, the article demonstrates how Bayesian updating enhances credibility in challenging valuations, improves data utilization, and better communicates risk to clients. This methodology offers appraisers a powerful tool that balances analytical rigor with practical application in both residential and commercial settings.

Introduction

For decades, real estate appraisers have relied on experience, intuition, and traditional valuation methods to determine property values. Yet, in a world where data and computing power are abundant, uncertainty remains. Much like Otis Redding's classic song "(Sittin' On) The Dock of the Bay," which captures the contemplative act of observing and reflecting, appraisers must synthesize historical market trends and new sales evidence to arrive at well-supported conclusions.

But what if, rather than passively observing the market, appraisers could formally update their beliefs about value in a structured way, incorporating both prior knowledge and new data as it comes in? This is precisely where Bayes' Theorem comes into play. Bayesian inference provides a principled framework for integrating expert judgment with evolving market evidence, allowing appraisers to refine their estimates dynamically rather than relying solely on static adjustments.

Why should real estate appraisers consider Bayesian methods? In today's complex and sometimes data-limited markets, Bayesian approaches offer several immediate practical advantages:

- **Enhanced credibility in challenging appraisals:** When comparable sales are limited or imperfect, Bayesian methods provide a rigorous framework for incorporating professional judgment alongside market evidence.
- **Risk-aware valuations:** By quantifying uncertainty, Bayesian appraisals provide clients with crucial risk information, particularly valuable in volatile markets or on unusual properties.
- **Defensible reconciliation:** Rather than subjective weighting of comparables, Bayesian updating provides a mathematical foundation for how different pieces of evidence contribute to the appraiser's final value conclusion.
- **Adaptability to data quality:** The Bayesian framework naturally gives more weight to high-quality data while still extracting value from imperfect comparables that might otherwise be discarded.
- **Continuous improvement:** As new market data emerges, Bayesian methods allow for systematic updating of valuations without starting the analysis from scratch.

This article explores how these advantages can be realized through practical application of Bayes' Theorem in the appraisal process.

Real estate appraisal plays a critical role in financial markets and lending decisions. The global financial crisis of the late 2000s underscored the importance of reliable property valuations and led to increased demands for objectivity and accuracy in appraisal practice. For example, regulations in some jurisdictions require that appraised values fall within tight ranges (e.g., within 20% of true market value).[1] Meeting such standards consistently is challenging when appraisals rely heavily on limited comparable sales and subjective adjustments.

Traditional appraisal methods, while grounded in professional expertise, often lack a formal statistical basis for handling uncertainty. Appraisers' judgment is highly respected and typically quite accurate, yet their expertise is not explicitly incorporated in the quantitative analysis of value. In standard practice, the sales comparison approach involves selecting a few comparable sales and adjusting their prices to estimate the subject property's value. However, these adjustments (e.g., for differences in size, location, or amenities) are usually applied deterministically and based on experience, without an accompanying measure of confidence or formal probabilistic reasoning. As a result, the final value estimate is presented as a point value with little indication of its uncertainty or the reasoning process behind the mathematical weighting of multiple comparable indicators except what may be written in the reconciliation of the approach or the appraisal.

Bayesian statistical methods offer a potential solution by providing a principled way to combine prior knowledge (expert judgment, historical data) with new evidence (recent sales) to arrive at a more robust estimate. Bayes' Theorem allows the appraiser to formally update their prior beliefs about value considering new sale information, yielding a posterior or subsequent estimate that reflects both sources of information. Researchers have noted that, unlike classical approaches, a Bayesian-style technique "easily integrates expert judgment and responds flexibly to the arrival of new information"[2] in valuation analyses. This means an appraiser's market expertise and intuition can be quantified as prior probabilities, then refined as hard data on comparables are introduced. The outcome is not only a potentially more accurate value estimate, but also an explicit quantification of uncertainty (through probability distributions or confidence intervals) around that estimate.

In the context of appraisal standards, this Bayesian updating process directly supports the objective of producing credible assignment results as required under USPAP Standard Rule 1-1. By systematically refining prior opinions of value with new, verifiable evidence, the appraiser demonstrates due diligence in applying recognized methods and integrating both qualitative judgment and quantitative data. This structured approach not only enhances analytical transparency but also aligns with the standard's emphasis on reasoning that can be clearly communicated and tested.

By leveraging Bayesian methods, appraisers can move beyond deterministic value conclusions and instead embrace probability-based reasoning to improve the accuracy, transparency, and defensibility of their valuations. This article explores how Bayes' Theorem can be applied to real estate appraisal, demonstrating how conditional probability, Bayesian updating, and confidence intervals can enhance traditional approaches to value adjustments, comparable selection, and uncertainty analysis. Through theoretical discussion and an illustrative example, this article shows how appraisers can transition from merely watching the market to actively integrating market data and helping them navigate valuation with a more structured, statistically rigorous approach. By integrating Bayesian methods, both residential and commercial appraisers can improve the credibility and defensibility of their valuations, especially in an era of increasing data availability and analytic scrutiny.

1. The German Federal Constitutional Court (Bundesverfassungsgericht) ruling from November 7, 2006 (1 BvL 10/02) established that the assessed value of property for inheritance tax purposes should not deviate more than ±20% from its market value. Bundesverfassungsgericht, "Leitsätze zum Beschluss des Ersten Senats vom 7. November 2006," November 7, 2006, https://bit.ly/3KES87o.

2. Nathan Berg, "A Simple Bayesian Procedure for Sample Size Determination in an Audit of Property Value Appraisals," *Real Estate Economics* 34, no. 1 (2006): 133–155.

Bayes' Theorem: Fundamental Concepts

Bayes' Theorem is a foundational principle of probability theory that provides a formal method for updating beliefs considering new evidence. It is "a mathematical formula for determining conditional probability."[3] In other words, it allows one to revise existing predictions or estimates when presented with additional data. In the context of real estate, Bayes' Theorem can be thought of as a tool that lets an appraiser update the estimated value of a property (the belief) after considering new information such as a recently observed sale (the evidence).

Mathematically, Bayes' Theorem is expressed as follows:

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

where:

- **A** represents the event or hypothesis—in an appraisal context, this might be a proposed value range for the subject property;
- **B** represents the observed evidence, such as comparable sales data;
- **P(A)** is the prior probability that the property's value lies within a given range before considering the new sales evidence;
- **P(B|A)** is the likelihood of observing that particular sales evidence if the value hypothesis (A) were true; and
- **P(B**) is the overall probability of observing the evidence, often termed the normalizing constant or "evidence."

Bayesian thinking in appraisal is like having an educated guess that gets refined with new evidence. Imagine you're appraising a three-bedroom home in a neighborhood where similar homes typically sell for around $350,000. This initial estimate, based on your general knowledge of the market, is your "prior belief." When you gather specific comparable data (like a similar home that sold for $375,000), Bayesian analysis helps you update your estimate. If the recent comparables are very similar to your subject property, they'll influence your final value opinion more heavily. The result is your "posterior belief" or a refined estimate that combines your professional experience with the specific evidence at hand.

Let me illustrate with a practical example: An appraiser is evaluating a 2,000-square-foot colonial home in a suburban neighborhood. Based on her experience and general market knowledge (her prior), she believes homes in this area typically value around $400,000 ($200 per square foot).

She then collects three comparable sales:

- Comparable 1: Very similar 2,100 sq. ft. colonial that sold last month for $430,000
- Comparable 2: Similar 1,900 sq. ft. colonial that sold two months ago for $410,000
- Comparable 3: Similar 2,000 sq. ft. colonial that sold three months ago for $415,000

This new evidence suggests that the value might be higher than her initial estimate. Using Bayesian thinking, she does not simply average these values or throw out her prior knowledge. Instead, she updates her belief by considering (1) how confident she was in her initial $400,000 estimate, (2) how similar these comparables are to the subject property, and (3) how recent and relevant the sales are. Her updated (posterior) estimate might be $415,000, giving more weight to the most recent and similar comparable while still incorporating her initial market knowledge.

The beauty of Bayesian thinking is that it formalizes how appraisers already think, starting with general market knowledge and refining it with specific evidence, rather than requiring previous appraisals of the exact same property.

The terms "prior" and "posterior" are central to Bayesian reasoning. The prior probability reflects our initial belief about an event or parameter before new data are considered.[4] It is essentially the appraiser's expectation of value based on experience, market trends, or earlier analysis, before examining the specific comparables at hand. The posterior probability is the updated belief after incorporating the new evidence; it is

3. "Bayes' Theorem Definition," Investopedia, accessed March 12, 2025, https://www.investopedia.com/terms/b/bayes-theorem.asp.

4. Adam Hayes, "Bayes' Theorem: What It Is, Formula, and Examples," Investopedia, accessed May 27, 2025, https://www.investopedia.com/terms/p/prior_probability.asp.

the probability of event A (e.g., a particular value range) given the observed data B.[5] Bayes' Theorem tells us how to turn the prior into the posterior by factoring in the likelihood of the evidence. Notably, the approach "relies on incorporating prior probability distributions in order to generate posterior probabilities,"[6] meaning that we explicitly model our prior knowledge as a distribution (rather than a single point estimate) and update it with data to get a new distribution of possible values.

Under the Bayesian paradigm, any unknown quantity such as the market value of a property is treated as a random variable rather than a fixed but unknown constant. In other words, it is important to acknowledge uncertainty by describing the property's value with a probability distribution, which represents degrees of belief in various possible values.[7] This is a fundamentally different viewpoint from the classical approach: rather than saying "the value is exactly $V, we just don't know it," the Bayesian approach says "based on what we know, the value has a probability X% of being around $V." As more information becomes available, this distribution can be updated and narrowed. Indeed, Bayes' Theorem "allows you to update the predicted probabilities of an event by incorporating new information."[8] This capability is precisely what makes Bayesian methods attractive for appraisal, as each new comparable sale or data point can be used to refine the value estimate in a logically consistent way.

By framing valuation in terms of probabilities, appraisers can derive not only a single most likely value (such as the mean or median of the posterior distribution), but also a measure of confidence in that estimate. The mechanics of Bayesian updating will be illustrated in later sections; first, let's turn to the concept of conditional probability, which underpins Bayes' Theorem and manifests in the practice of adjusting comparable sales.

Conditional Probability in Property Value Adjustments

At its core, the process of adjusting comparable sales is an exercise in conditional reasoning. Appraisers often ask: "Given that a comparable property sold for $X, what does that imply about the value of the subject property, considering their differences?" This is fundamentally a question of conditional probability. In probabilistic terms, we are interested in the distribution of the subject property's value conditional on the information provided by a comparable sale. Every adjustment an appraiser makes, be it for location, size, age, or amenities, implicitly answers a conditional question: If the comparable had been more like the subject (for example, if it did not have a swimming pool, or had the same square footage as the subject), then what price would it likely have sold?

Consider a simple scenario: a comparable property with a swimming pool sells for $330,000, while the subject property is identical except that it has no pool. The appraiser might estimate that the pool contributes, say, $20,000 to value in that market. In making this adjustment, the appraiser is effectively assessing the subject's likely selling price *given* the sale of a similar property at $330,000 with a pool. In notation, we might be interested in something like: $P(V_{SUBJECT} \approx \$310{,}000 \mid \text{Comp sale} = \$330{,}000, \text{Comp has pool})$.

The reasoning is that if we know a house with a pool sold at a certain price, the probability is high that a similar house without a pool would sell for around a correspondingly lower price (in this case, around $310,000). This conditional probability perspective formalizes the idea that the comparable's price informs the subject's value after accounting for specific differences.

In traditional appraisal practice, such adjustments are usually deterministic (e.g., subtract $20,000 for the pool). However, there is uncertainty around that adjustment. Perhaps the pool's

5. *Ibid*.
6. *Ibid*.
7. Lee Changro and Park Keith Key-Ho, "Representing Uncertainty in Property Valuation Through a Bayesian Deep Learning Approach," *Real Estate Management and Valuation*, vol. 28 (December 2020): 15–23.
8. "Bayes' Theorem Definition," Investopedia.

value could reasonably be $15,000 or $25,000 depending on buyer preferences. By thinking in terms of conditional probability, we acknowledge a range of possible "adjusted prices" for the subject given the comparable's sale. For instance, we might say there is an 80% chance that, given the comparable's $330,000 sale with a pool, the subject is worth between $290,000 and $330,000, and a smaller chance it could be outside that range.

Table 1 illustrates how probability intervals can be interpreted in appraisal practice. An 80 percent credible interval of ± $20,000 around the mean implies that most of the plausible market values for the subject lie between $290,000 and $330,000, acknowledging both professional judgment and data uncertainty.

Table 1. Probability Intervals and Value Ranges

Interval	Probability	Value Range
± $10,000	68% (≈ 1σ)	$300,000 – $320,000
± $20,000	80%	$290,000 – $330,000
± $30,000	95% (≈ 2σ)	$280,000 – $340,000

This way of thinking is precisely what Bayes' Theorem facilitates. In Bayesian terms, we would treat the subject's true value as uncertain and use the comparable's sale as evidence to update the probability distribution of the subject's value. The "likelihood" portion of Bayes' formula, $P(B|A)$, corresponds to how likely it is to observe the comparable's sale price B assuming a hypothesis A about the subject's value. If the comparable is very similar to the subject, then observing its sale price provides strong, tight information about the subject's value. The conditional distribution of the subject's value (given that sale) will be narrowly centered around the comparable's price. Conversely, if the comparable has significant differences requiring large adjustments, then the subject's value given that comparable sale is much more uncertain (a wider conditional distribution). In effect, each comparable sale can be viewed as defining a conditional probability distribution for the subject's value.

Embracing this viewpoint encourages appraisers to think of comparable adjustments not as fixed additions or subtractions, but as probabilistic statements. For example, instead of simply stating "Comparable A is $10,000 superior to the subject," a Bayesian-minded appraiser acknowledges something like: "Given Comparable A's sale, there is a high probability the subject's value is about $10,000 less, but there is a margin of error around that figure." Such an approach lays the groundwork for a more formal Bayesian updating process, wherein each comparable's conditional contribution to the subject's value is weighted according to its reliability. In the next section, this article expands on this concept by showing how Bayesian inference can be used to combine multiple comparables and prior information to refine the value estimate.

Incorporating probability bands into reconciliation encourages appraisers to communicate uncertainty more transparently. Instead of a single deterministic figure, the appraiser can report a most probable value along with an interval that captures the likely range of market outcomes. This approach not only improves analytical clarity but also aligns with USPAP's emphasis on producing credible results that reflect both market evidence and the appraiser's informed judgment.

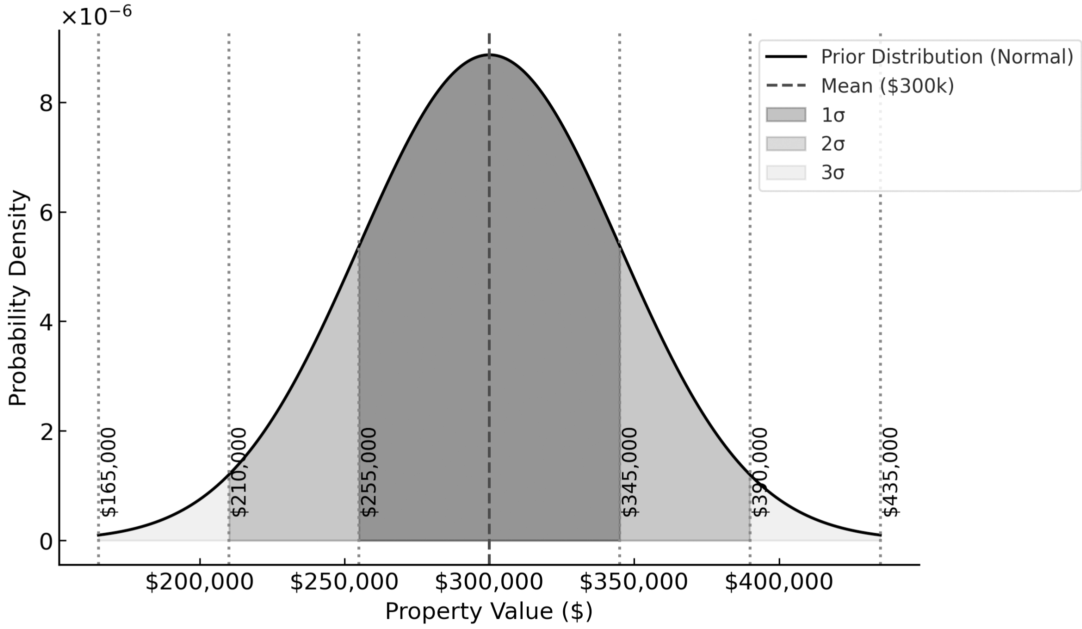
Refining Value Estimates with Bayesian Inference

The true power of Bayes' Theorem in appraisal emerges when combining multiple pieces of evidence such as several comparable sales along with any prior market knowledge the appraiser brings. The Bayesian approach provides a coherent mechanism to "update" an initial value estimate (prior) with information from each comparable (likelihood), resulting in a refined posterior estimate. In practical terms, this means the appraiser can start with an initial value distribution for the subject property and then sequentially incorporate each comparable sale, each time sharpening the estimate.

A Bayesian valuation process might be implemented in four general steps:

1. **Establish a prior.** Define a prior distribution for the subject property's value. This could be based on an initial appraisal judgment, an automated valuation model, or broad market statistics. For example, before examining comparables, an appraiser might believe the subject is likely worth around $300,000 ± 15% (reflecting substantial uncertainty). This belief is encoded as a probabil-

Figure 1. Prior Distribution for Property Value



ity distribution (e.g., a normal distribution centered on $300,000), as shown in Figure 1.

2. Define likelihoods from comparables. For each comparable sale, formulate a likelihood function for the subject's value. In simpler terms, determine how informative that sale is about the subject. This involves adjusting the comparable's sale price for differences and assessing the uncertainty of that adjusted value. For instance, a very similar comparable might be assumed to indicate the subject's value within ±5% (small error), whereas a less comparable sale (perhaps in a different neighborhood or older transaction) might have ±10% or more uncertainty. These uncertainties can be represented by a variance or standard deviation for each comparable's indication.

3. Update to posterior. Apply Bayes' Theorem to update the prior with the information from the comparables. This can be done analytically if we assume normal distributions (a convenient case where the mathematics has a closed-form solution), or through computational methods such as Markov Chain Monte Carlo simulation[9] for more complex cases. Conceptually, each comparable "shifts" and refines the prior distribution. After processing all comparables, the result is a posterior distribution for the subject's value that reflects both the initial knowledge and all the sale evidence.

4. Analyze the posterior estimate. The posterior distribution represents the final belief about the subject's value after considering the comparables. From this distribution, the appraiser can extract a point estimate (for example, the mean or median as the most likely value) and a credible interval (for example, a range in which the value lies with 95% probability, as discussed later). The posterior's spread (standard deviation) indicates the remaining uncertainty: ideally, it will be narrower than the prior spread, signifying increased confidence due to the market evidence.

One intuitive outcome of this Bayesian updating is that the more a comparable resembles the subject property, the more heavily it is weighted in the final estimate, a feature that appraisers will recognize from traditional weighting of comparables. In fact, under the common assumption of normally distributed errors, the Bayesian posterior mean turns out to be a weighted average of

9. A. N. Sanborn, "Markov Chain Monte Carlo," in M. C. Frank and A. Majid (Eds.), Open Encyclopedia of Cognitive Science. (MIT Press, 2024), https://doi.org/10.21428/e2759450.56eb7cff.

the prior estimate and the comparables' adjusted values, where the weights are proportional to their precision (inverse variance).[10] In other words, a very reliable data point (low variance) will heavily influence the result, whereas a noisy data point (high variance) will have a smaller impact.[11] This matches appraisal intuition: if one comparable is almost identical to the subject, an appraiser gives it more weight, whereas a less comparable sale is given less weight. Bayesian inference formalizes this process.

The benefits of this refinement are evident. If the initial prior was broad, say the appraiser was very unsure, or the market had not been analyzed yet, incorporating a few reliable comparables will substantially narrow the value range. On the other hand, if the prior knowledge was strong (imagine a highly experienced appraiser or very pertinent previous data), the method won't overrule it unless the comparable evidence is convincingly different. The comparables will adjust the estimate only to the extent they provide new information. This dynamic balance is a strength of Bayesian analysis: it "easily integrates expert judgment and responds flexibly to the arrival of new information"[12] as noted in prior research. It also means that in data-sparse situations, the appraiser's informed prior prevents the estimate from flailing with too little data, while in data-rich situations, the prior is naturally overshadowed by market evidence yielding results akin to a traditional sales comparison or regression analysis.

Studies in real estate valuation have demonstrated the efficacy of such Bayesian refinement. For example, Weitkamp and Alkhatib incorporated expert opinion as priors in a Bayesian valuation model for residential properties and found that the posterior results yielded "improved mean values… and more certain accuracy"[13] in the estimated coefficients of value factors. In other words, by blending an appraiser's expertise with market sales data, the estimates of how different property characteristics affect value became both more accurate and more statistically reliable than those from a purely data-driven model or purely judgment-based approach alone.

A study published in the *International Real Estate Review* developed a statistical method for residential property valuation using a hierarchical Bayesian approach. In simple terms, a hierarchical Bayesian approach allows the model to learn at more than one level. It treats each property's valuation as part of a broader market structure wherein individual homes are modeled within neighborhoods, and neighborhoods within the larger market. The researchers found that their model outperformed other valuation methods, including those based on average price-per-square-feet or expert assessments.[14] This kind of outcome is directly analogous to an individual property appraisal: a Bayesian approach can improve the accuracy of the final value estimate and provide quantified confidence in that estimate.

In the following section, we address how uncertainty in data and market conditions can be systematically analyzed within a Bayesian framework, and how the results can be communicated through confidence intervals (credible intervals) for the value opinion.

Dealing with Uncertainty and Limited Data

Uncertainty is an inherent aspect of real estate valuation. Market conditions fluctuate, comparable data can be sparse or imperfect, and every appraisal involves some degree of professional judgment. Traditional appraisal reports, however, seldom make this uncertainty explicit. Appraisers typically provide a single point value and at most discuss qualitatively if the market is "stable" or "volatile." An experienced appraiser might internally think, "the value is around $500,000, give

10. Robert Jacobs, "Bayesian Statistics: Normal-Normal Model" (working paper, Department of Brain & Cognitive Sciences, University of Rochester, December 3, 2008).

11. *Ibid*.

12. Nathan Berg, "A Simple Bayesian Procedure for Sample Size Determination in an Audit of Property Value Appraisals," 133–155.

13. Alexandra Weitkamp and Hamza Alkhatib, "The Bayesian Approach in the Valuation—A Strategy to Handle Markets with Low Purchasing Prices?" (Germany), accessed March 12, 2025, https://bit.ly/49Vot4m.

14. Sam K. Hui, Alvin Cheung, and Jimmy Pang, "A Hierarchical Bayesian Approach for Residential Property Valuation: Application to Hong Kong Housing Market," *International Real Estate Review* 13, no. 1 (2010): 1–29.

or take perhaps 10%," but this margin is not formally reported. (As noted earlier, some valuation standards implicitly acknowledge uncertainty, for example, German regulations tolerate up to a 20% deviation in appraised values, yet the appraisal process itself does not quantify that uncertainty for the end user.

A Bayesian approach compels us to confront uncertainty head-on. By modeling the property value as a distribution and the comparables as noisy indicators, we naturally obtain a measure of how uncertain the estimate is. If the available data are scant or not entirely reliable, the posterior distribution will be wider, signaling less confidence in the conclusion. On the other hand, if data are plentiful and consistent, the posterior will be narrow. This feature is valuable in practice: it prevents overconfidence when evidence is weak, and it highlights when additional data or investigation might be needed. For instance, unusual market conditions, such as COVID-era GSE flexibilities, a sudden interest rate hike, or a natural disaster, mean that recent sales are not perfectly reflective of the subject's value. A Bayesian model can incorporate that insight as part of the prior or likelihood, perhaps by increasing the assumed variability of sale prices. The result would be a more cautious (wider) value range, appropriately reflecting the uncertain environment, rather than a false veneer of precision with a single point estimate.

Bayesian methods also enable the use of all available information, even if some of it is imprecise. In a conventional appraisal, one might exclude a somewhat dated sale or a sale from a neighboring town because it is not a perfect comparable. In a Bayesian framework, that sale can still inform the valuation, just with a larger uncertainty attached to it. The method will down-weight its influence according to the evidence's relevance and reliability. This is preferable to ignoring potentially useful data altogether. As an example, imagine appraising a highly unique property with only one directly comparable sale in the past year. A traditional approach might rely almost entirely on that one comparable and likely a lot of judgment, whereas a Bayesian approach could start with a wider prior gleaned from broader market trends or similar properties in other areas, and then update with the one local comparable. The outcome would be an estimate that acknowledges the large uncertainty due to limited data but still makes the best use of what is known.

Research demonstrates the value of quantifying uncertainty. In a study using machine learning for house price prediction, Lee and Park found that a Bayesian neural network model did not necessarily yield lower prediction error than a traditional model, but it did "provide a tool to measure the uncertainty inherent in predicted prices."[15] In other words, the Bayesian approach was able to not only forecast value but also give a confidence measure for each prediction, something crucial for risk management. Their findings underscore that a "Bayesian approach can model uncertainty in property valuation,"[16] adding transparency for end-users who want to know the risk around a point estimate. The prior referenced study by Hui et al. applied a hierarchical Bayesian model to residential property values and reported that this approach outperformed simpler valuation methods based on average price-per-square-foot or even expert appraisals.[17] The Bayesian model's ability to incorporate multi-level data such as neighborhood-level trends and individual property characteristics allowed it to handle data complexities and uncertainty more effectively than the crude heuristics, ultimately improving accuracy.

Bayesian inference can even improve decision-making about data collection itself. For example, in the realm of assessment oversight, Berg (2006) showed that a Bayesian auditing technique "significantly reduces the number of reappraisals required"[18] when evaluating the accuracy of property tax appraisals. By continuously updating the estimated error rates as new audit results came in, the procedure could determine when enough evidence had been gathered to be confident that val-

15. Lee Changro and Park Keith Key-Ho, "Representing Uncertainty in Property Valuation Through a Bayesian Deep Learning Approach," 15–23.

16. *Ibid*.

17. Sam K. Hui, Alvin Cheung, and Jimmy Pang, "A Hierarchical Bayesian Approach for Residential Property Valuation: Application to Hong Kong Housing Market," 1–29.

18. Nathan Berg, "A Simple Bayesian Procedure for Sample Size Determination in an Audit of Property Value Appraisals," 133–155.

uations were within tolerance, thereby avoiding unnecessary additional audits. This finding illustrates a broader point: understanding and modeling uncertainty can lead to more efficient allocation of effort in valuation work. In everyday appraisal practice, this might translate to knowing when the available data are sufficient versus when more comparables or analysis are needed to reach a reliable conclusion.

In summary, treating property valuation as a probabilistic problem helps appraisers deal with uncertainty in a systematic way. Rather than relying on gut feelings about a “cushion” or using overly conservative estimates to hedge against unknowns, the Bayesian framework makes uncertainty an explicit part of the analysis. The outcome is a more informative appraisal and one that not only provides the best estimate of value but also communicates how much confidence one can have in that estimate based on the data at hand. One effective way to communicate this confidence (or uncertainty) to clients and stakeholders is using confidence intervals.

Limitations and Assumptions

While Bayesian methods strengthen the statistical foundation of appraisal, they are not immune to the limitations of underlying data and model assumptions. The reliability of any Bayesian estimate depends on the accuracy of the prior beliefs and the quality of comparable sales data used to update them. In practice, priors may incorporate professional judgment, which introduces subjectivity even as it formalizes experience. Furthermore, when multiple variables interact such as location, financing conditions, and property quality, the computational complexity increases, and simplifications must often be made. Appraisers should therefore treat Bayesian results as informed estimates that complement, but do not replace, traditional reasoning. The most credible applications are those that balance analytical rigor with professional judgment and transparently acknowledge uncertainty in both assumptions and outcomes.

The process of assigning probabilities involves professional judgment that varies among appraisers, meaning that two practitioners with different experience or market interpretation could derive slightly different results. Additionally, real estate markets are complex, involving interrelated factors such as location, physical condition, financing, and timing; modeling all these simultaneously can become computationally challenging. Finally, Bayesian models are sensitive to the quality of new evidence. A non-arm’s-length sale or atypical transaction can skew results unless identified and down-weighted through proper data vetting. Recognizing these constraints is essential to applying Bayesian methods responsibly and transparently.

Confidence Intervals for the Final Value Estimate

One of the most valuable outputs of a Bayesian appraisal analysis is a natural quantification of confidence in the final value estimate. Rather than simply stating, for example, “The market value is $305,000,” the appraiser could report something like: “The analysis indicates a most probable value of $305,000, with a 95% confidence interval of $290,000 to $320,000.” This means that, given the data and assumptions, there is a high probability that the true market value lies within that range. Such a statement is far more informative to a client or underwriter than a single number alone.

It is important to clarify terminology: in classical statistics one would speak of a “95% confidence interval,” whereas in Bayesian analysis one would more properly speak of a “95% credible interval.”[19] The concepts are similar in that both provide a range likely to contain the true value, with the Bayesian credible interval having the straightforward interpretation of a 95% probability given the evidence that the true value lies in that range. For the purposes of appraisal reporting, we can use the term confidence interval in a general sense understood by most readers. The key is that we convey a range and a level of confidence, which communicates the uncertainty inherent in the estimate.

Including a confidence interval in an appraisal has several benefits:

19. Ruslan Shudra, “Understanding Confidence Intervals vs Credible Intervals in Data Analysis,” Codefinity (blog), January 2024, https://codefinity.com/blog/Understanding-Confidence-Intervals-vs-Credible-Intervals-in-Data-Analysis.

- **Enhanced transparency:** It openly conveys the degree of uncertainty. This can prevent misinterpretation of the appraisal as being more precise than the data support. For example, if the interval is wide, a client understands that the market evidence was limited or volatile, whereas a narrow interval signals a high degree of agreement among the data.
- **Risk management:** Users of appraisals, such as lenders and investors, can make more informed decisions. A lender might be more cautious or require additional collateral or a lower loan-to-value ratio if the 95% confidence interval for a property's value spans a wide range, as this indicates higher risk. Conversely, a tight interval could give additional comfort about the collateral's value stability.
- **Appraiser accountability and credibility:** Presenting an interval demonstrates that the appraiser has conducted a thorough analysis of uncertainty. This level of analytical thoroughness can add credibility, especially in litigation or review scenarios where the strength of evidence is scrutinized.

It is worth noting that the idea of reporting uncertainty is not entirely foreign to real estate professionals. In mass appraisal such as for tax assessment purposes, statistical measures are routinely reported. For instance, assessment offices often calculate confidence intervals around the median sales ratio to test whether valuations meet required standards.[20] This is analogous to saying, "we are 95% confident the median assessment level is within a certain range of the market value." However, in single-property appraisal reports for lending or investment, such statistical reporting has not been standard practice. Bayesian analysis provides a framework to change that, by making it straightforward to derive an interval estimate as part of the valuation process.

In practice, an appraiser could choose a confidence level, commonly 90% or 95%, for the interval to report. A 95% interval is wider (more conservative) than a 90% interval, as it reflects greater confidence. The chosen level might depend on client preferences or industry norms. Regardless, the process involves extracting the appropriate quantiles from the posterior distribution of value. In our earlier illustrative example, if the posterior distribution of the subject's value had a mean of $307,000 and a standard deviation of about $7,000, the 95% credible interval was approximately $307,000 ± $13,000 (roughly $294,000 to $320,000).[21] Reporting this informs the client that, given the available data, the subject property's value is not pinpointed to a single figure but lies within a band of likely outcomes.

By providing a confidence interval, appraisers can communicate the uncertainty in a pragmatic way, much as other fields do when presenting statistical results. This practice could improve understanding between appraisers and clients: rather than debating whether an appraisal is "right or wrong," the conversation can shift to the level of certainty in the valuation and what might reduce the uncertainty such as additional data, more research, etc. In fast-changing markets or cases with limited comparables, this could be especially useful. Ultimately, the inclusion of confidence intervals turns an appraisal report into a richer informational tool, helping stakeholders make better-informed real estate decisions.

Illustrative Example: Bayesian Valuation of a Subject Property

To demonstrate the earlier concepts, consider a simplified appraisal scenario for a fictional residential property. Suppose the subject property is a 2,000-square-foot single-family home, built in 2000, without a swimming pool. Based on prelim-

20. Colorado Department of Local Affairs, "Chapter 8 - Statistical Measurements," Assessors' Reference Library, accessed March 12, 2025, https://arl.colorado.gov/chapter-8-statistical-measurements. Also, International Association of Assessing Officers, "Standard on Ratio Studies," April 2013, https://www.iaao.org/wp-content/uploads/Standard_on_Ratio_Studies.pdf.

21. In a normal distribution, a 95% confidence interval is typically calculated as: Mean ± (1.96 × Standard Deviation). Given the information provided: Mean = $307,000; Standard Deviation = $7,000. Calculating the 95% confidence interval: $307,000 ± (1.96 × $7,000) $307,000 ± $13,720. This rounds to approximately $13,000, resulting in the interval from $294,000 to $320,000. The multiplier 1.96 represents the z-score for a 95% confidence level in a normal distribution. If we used exactly 2 standard deviations (which would give ±$14,000), that would correspond to a confidence level of about 95.4%, slightly higher than the standard 95%. The reported $13,000 is therefore for a typical 95% confidence interval calculation, with slight rounding applied.

inary analysis (or perhaps an AVM output), the appraiser's prior belief is that the home's value is around $300,000, but this is a very uncertain estimate (the appraiser is only, say, 50% confident the value lies between $280,000 and $320,000). We will treat this prior belief as a normal distribution: mean = $300,000 and a large standard deviation = $30,000, reflecting the broad ±$59,000 range for 95% of the probable values.

Next, the appraiser identifies three comparable sales (Table 2) to use as evidence. These comparables are simplified for this example:

Table 2. Comparable Sales Data

Property	Sale Price	Living Area (SF)	Year Built	Has Pool?
Subject	- (n/a)	2,000	2000	No
Comp. A	$300,000	2,100	1995	No
Comp. B	$310,000	1,900	2005	No
Comp. C	$330,000	2,000	2000	Yes

Data for illustrative purposes

Comparable A is slightly larger in size than the subject (100 SF more) but older in age; Comparable B is slightly smaller (100 SF less) but newer; Comparable C is essentially identical in size/age but it features a swimming pool that the subject lacks. To adjust these comparables, the appraiser assumes that size is worth about $50 per square foot and that a pool contributes roughly $20,000 in this market (these figures are hypothetical). Applying these adjustments:

- **Comparable A:** 100 SF larger means if it were the same size as the subject, its sale price might be about $5,000 lower. Being older might have an opposite effect, perhaps adding some value back, but for simplicity we assume the size effect dominates. The adjusted indicated value from Comparable A for the subject is approximately $295,000.
- **Comparable B:** 100 SF smaller means if it were upsized to the subject's 2,000 SF, its price might be about $5,000 higher. After that adjustment, Comparable B suggests roughly $315,000 for the subject.
- **Comparable C:** This property's pool gave it a higher sale price. Removing the pool's influence by subtracting $20,000 yields an indicated value of about $310,000 for a pool-less property like the subject.

Now we have three adjusted indications: $295,000, $315,000, and $310,000. The appraiser could reconcile these by some weighted average, but here we will use Bayesian updating to formally combine them with the prior. We treat each comparable's indicated value as an observation with a certain uncertainty. Comparable A, being a bit older and larger, is less directly comparable; therefore, we assign it a wider error band (say ±$15,000) to reflect that. Comparable B is quite similar (just slightly smaller/newer), so we give it a tighter expected error (±$10,000). Comparable C is very similar except for the pool, so we assign it an intermediate uncertainty (±$12,000). (See Table 3.) These error estimates serve as the standard deviations for the likelihood functions of each comparable. Using Bayes' theorem, we update the prior distribution with each comparable sequentially. As a note, in practice, the order doesn't matter; the result will be the same regardless of sequence, assuming all comparables are independent evidence.

After updating all three comparables, the resulting posterior distribution for the subject's value has a mean of approximately $309,000 and a standard deviation of about $7,000 (computed via an R script implementing the Bayesian update).[22] Table 4 outlines the detailed confidence interval calculations.

Table 5 summarizes the prior and posterior estimates side by side. As shown, the posterior estimate of value is around $309,000, which is slightly higher than the initial guess, because two of the comparables indicated values above $300,000 (thus pulling the estimate upward). More impor-

22. See the appendix for the R code used and sample simplified Python code example. Both R and Python are open-source programming languages freely accessible by appraisers. I used R but this can also easily be performed in Python. For more complex Bayesian models (hierarchical models, non-conjugate priors, complex likelihood functions, etc.), you would typically need specialized libraries that implement MCMC (Markov Chain Monte Carlo) methods like Stan, JAGS, or BUGS through R packages like "rstan," "brms," or "rjags." This example is simplified for understanding and typical code for the relatively simple calculations.

Table 3. Comparable Adjustments and Uncertainties

Comparable	Original Price	Adjustment	Adjusted Value	Uncertainty(±)
Comp. A	$300,000	–$5,000	$295,000	$15,000
Comp. B	$310,000	$5,000	$315,000	$10,000
Comp. C	$330,000	–$20,000	$310,000	$12,000

Table 4. Detailed Confidence Interval Calculations: Step-by-Step Progression of Bayesian Updating

Step	Mean	SD	Z (95%)[1]	CI Calculation[2]	95% Confidence Interval	Range(±)[3]
Prior	$300,000	$30,000	1.96	Mean ± Z x SD = $300,000 ± $59,000	$241,000 – $359,000	±$59,000
After Comp. A	$296,000	$13,000	1.96	Mean ± Z x SD = $296,000 ± $26,000	$270,000 – $322,000	±$26,500
After Comp. B	$308,000	$8,000	1.96	Mean ± Z x SD = $308,000 ± $16,000	$292,000 – $324,000	±$15,500
After Comp. C	$309,000	$7,000	1.96	Mean ± Z x SD = $309,000 ± $13,000	$296,000 – $322,000	±$13,000

1. Z-score of 1.96 corresponds to 95% confidence level in a normal distribution
2. The confidence interval calculation follows the formula: Mean ± (Z × Standard Deviation)
3. Range(±) shows the plus/minus value from the mean, rounded to nearest $1,000

Table 5. Prior vs. Posterior Value Estimate

Estimate	Mean Value	Standard Deviation	95% Confidence Interval
Prior (Initial)	$300,000	$30,000	$241,000 – $359,000
Posterior (Updated)	$309,000	$7,000	$296,000 – $322,000

tantly, the uncertainty has shrunk dramatically: the prior range of ±$59,000 (a 20% uncertainty in either direction) has narrowed to about ±$13,000 (roughly 4% of the value). In other words, after considering the evidence from these sales, we are much more confident about what the subject property is worth. The 95% credible (confidence) interval of $296,000 to $322,000 provides a clear expression of this confidence. An appraiser could report this range to convey the uncertainty, as discussed earlier. If additional comparables or data were available, they could further refine the estimate (for instance, adding another very similar sale might tighten the interval even more around, say, $305,000 to $315,000). Conversely, if the comparables had been highly inconsistent with each other, the posterior distribution might not have narrowed so much, which would be reflected in a wider interval and would signal to the appraiser and the client that the valuation comes with greater uncertainty.

This example, although simplified, highlights how Bayes' theorem can be applied in appraisal. The prior encapsulated the appraiser's general market knowledge, and the data from the comparable sales were used to update that knowledge in a structured way. The result was a value conclusion that honors both the initial expectations and the actual market evidence and quantifies the uncertainty in the conclusion. In a real-world application, an appraiser could follow a similar process with more rigorous determination of priors and likelihoods. The outcome is a more nuanced valuation: not just "the property is worth $309,000," but "the property is most likely worth around $309,000, and the reasonable range of value given the current market evidence is roughly $296,000 to $322,000." By expressing results as a credible interval rather than a single point estimate, the appraiser communicates both precision and uncertainty, providing a more transparent and defensible basis for the final value conclusion.

Addressing Practical Implementation Concerns

Introducing Bayesian methods to clients accustomed to traditional appraisal reports requires a thoughtful approach. For lender clients, consider a gradual introduction of uncertainty measures. Start by including a brief "Market Conditions and Value Stability Analysis" section in your reports where you can discuss value ranges in qualitative terms: "Based on current market evidence, the concluded value demonstrates high stability with an estimated variation of less than ±5%." This introduces the concept without altering the familiar format of the report.

For sophisticated clients such as investors or portfolio managers, you might offer a supplemental "Value Risk Analysis" that includes credible (confidence) intervals: "While the most probable value is $309,000, our analysis indicates a 95% probability that market value falls between $296,000 and $322,000." Frame this as additional insight rather than uncertainty about your conclusion.

When presenting to clients, emphasize that expressing value ranges represents greater analytical rigor and transparency, not less confidence in your work. Use language like "probability distribution" rather than "uncertainty" to convey statistical sophistication rather than doubt. As with any analytical tool, the method's credibility depends on the appraiser's skill in specifying reasonable priors and identifying reliable comparables.

Regulatory Considerations

Bayesian approaches complement rather than conflict with existing appraisal standards and regulations. The Uniform Standards of Professional Appraisal Practice (USPAP) emphasizes that appraisers must develop credible opinions through appropriate methods. Specifically, Standards Rule 1-1(c) states that an appraiser must "not render appraisal services in a careless or negligent manner, such as by making a series of errors that, although individually might not significantly affect the results of an appraisal, in the aggregate affects the credibility of those results". Bayesian analysis enhances this credibility by providing a more rigorous statistical foundation for the opinion of value.

USPAP Standards Rule 2-2 requires that reports "contain sufficient information to enable the intended users to understand the report properly." Bayesian confidence intervals can be included as supplementary analysis that increases transparency without contradicting the requirement for a specific opinion of value. The reconciliation section of a report provides an ideal place to discuss how different comparables were weighted using Bayesian principles to arrive at the final value.

International Valuation Standards (IVS) similarly emphasizes transparency in methodology and recognize that different valuation approaches may yield a range of values. The Bayesian framework aligns with this recognition of value uncertainty while providing a formal structure to analyze it.

Conclusion

Bayesian methods, and Bayes' Theorem in particular, offer a powerful augmentation to traditional real estate appraisal techniques. By embracing statistical reasoning, appraisers can turn subjective adjustments and isolated sales data into a cohesive analysis that combines prior expertise with market evidence. In doing so, the appraisal process becomes more transparent and defensible: every adjustment is part of a logical probabilistic update, and the final value opinion comes with an understanding of how certain or uncertain this opinion is.

For both residential and commercial appraisers, the approaches discussed, from applying conditional probabilities in comparable adjustments to developing full Bayesian valuation models, are tools that can enhance practice. They are especially useful in situations with limited data or atypical properties, where standard approaches struggle, and uncertainty is high. Even in routine appraisals, Bayesian techniques can bring increased objectivity by systematically weighting evidence and insight by revealing the confidence level of conclusions. The use of credible (confidence) intervals and probability distributions moves the appraisal report from a single-point assertion to a richer informational product, aligning valuation with the risk-aware perspective that investors and lenders often have.

Implementing Bayesian appraisal does require a shift in mindset and the use of statistical software or models. Appraisers might start by incorporating interval estimates in their reports or by using simple Bayesian weighting formulas to reconcile comparables. With more accessible computing tools (many of which, like R or Python-based libraries, are readily available), even complex Bayesian calculations like Markov Chain Monte Carlo simulation can be performed to handle non-normal situations or multiple correlated factors. Notably, the added rigor does not replace the appraiser's judgment, it complements it. As the case example illustrated, an appraisal conducted with Bayesian inference yields a value estimate anchored in both experience and evidence, complete with an expression of uncertainty. This can improve client understanding and trust, as clients are made aware of the valuation's robustness and the range of plausible outcomes. In an age where data analytics and probabilistic forecasting are becoming the norm in many industries, incorporating Bayes' theorem into appraisal practice is a natural evolution.

Just as Otis Redding watched the tide roll away appraisers have traditionally observed market fluctuations from a distance. But with Bayesian methods, they need no longer be passive observers. Instead of "wasting time" watching market data drift by, appraisers can now systematically incorporate each new piece of evidence, updating their value distributions with mathematical precision. In the evolving profession of real estate appraisal, those sitting on the dock of the Bayes will find themselves not just watching the market tide but navigating it with newfound confidence and clarity.

In conclusion, Bayes' theorem is more than just a formula from a statistics textbook. It is a practical framework for reasoning under uncertainty. By adopting these methods, appraisers can enhance their analyses, better quantify the uncertainties they grapple with, and ultimately deliver value conclusions that are both well-supported and meaningfully qualified. This not only elevates the scientific rigor of appraisal but also reinforces its professional credibility in the eyes of clients, regulators, and the market at large.

About the Author

Jim Amorin, MAI, SRA, AI-GRS, CDEI, CAE, ASA, has been engaged in the real estate appraisal and consulting arena since 1988. Amorin authored the seminal text *The Generative Shift: Preparing Appraisers for Artificial Intelligence Models like ChatGPT* (2024). He has developed and reviewed numerous seminars for the Appraisal Institute, including *Market Analysis and the Site to Do Business*, *What Clients Want*, *Advanced Spreadsheet Modeling for Valuation Applications*, *Using Spreadsheet Programs in Appraisals*, *The Lending World in Crisis*, and *Introduction to Valuing Commercial Green Buildings*. He received the Appraisal Institute President's Award for his work related to the Institute's Capstone programs. He holds a bachelor of business administration degree in finance and a bachelor of business administration degree in real estate and urban land development from the University of Texas at Austin. He has testified before the US Congress and presented to the United Nations on real estate valuation, and he has served as a resource in the national press for stories in *The New York Times*, *National Mortgage Professionals*, *Housing Wire*, CNBC, CNN, *The Wall Street Journal*, and many other publications. He has taught courses and seminars and provided lectures in forty states and numerous countries/territories including Italy, Germany, Portugal, the People's Republic of China, Saudi Arabia, United Arab Emirates, Romania, Japan, Mexico, and Canada. He served on the national board of directors for the Residential Energy Services Network (RESNET) from 2017 through 2023. Amorin served as national president of the Appraisal Institute in 2009 and 2017 and as the Appraisal Institute chief executive officer from November 2017 through February 2023. **Contact: jamorin@me.com**

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Additional Resources

Suggested by the Y. T. and Louise Lee Lum Library

Appraisal Institute

- **Lum Library, Bibliographies [Login required]**
  - Bayesian analysis [Illumin Knowledge Base - 10944] |
  - Subjects: | Bayesian analysis; Statistical analysis (75); Data, statistics, and statistical analysis (13)

- **Publications—*Appraisal Journal* articles**
  - George Dell, "Regression, Critical Thinking, and the Valuation Problem Today" (2017)
  - Christian Schutzenhofer, Wolfgang Brunauer, Gerrit Leopoldsberger, Sven Bienert, and Kerstin Bobsin, "Energising Property Valuation: Putting a Value on Energy-Efficient Buildings [International Appraising]" (2011)
  - Bruce R. Weber, "A Beginning Best Practice Brownfield Valuation Model" (2002)
  - Richard J. Curcio, James P. Gaines, Richard E. Bennett, and James R. Webb. "Bayesian Regression Procedures Applied to the Valuation of Residential Real Estate" (1981)
  - Paul A. Howcroft, "A Review of Appraisal Practices and Methods of Analysis" (1978)
  - James C. Paddock, "California Orchard and Vineyard Appraisals" (1968)

Appendix 1. Sample R Code

```
Sample R Code
# Load required packages
library(gt)
library(dplyr)
library(scales)

# Set options to avoid scientific notation
options(scipen = 999)

# Set random seed for reproducibility
set.seed(123)

# Prior distribution parameters
prior_mean <- 300000
prior_sd <- 30000

# Comparable sales data after adjustments
comp_values <- c(295000, 315000, 310000)  # Adjusted values
comp_sds <- c(15000, 10000, 12000)        # Uncertainties for each comp

# Create a data frame to track the Bayesian updating process with detailed calculations
calculation_steps <- tibble(
      Step = c("Prior"),
      Mean = c(prior_mean),
      SD = c(prior_sd),
      `Z (95%)` = c(1.96),
      `Z × SD` = c(1.96 * prior_sd),
      CI_Lower = c(prior_mean - 1.96 * prior_sd),
      CI_Upper = c(prior_mean + 1.96 * prior_sd),
      `CI Range` = c(2 * 1.96 * prior_sd)
)

# Function to perform Bayesian update with one comparable
bayesian_update <- function(prior_mean, prior_var, obs_mean, obs_var) {
      # Calculate posterior parameters using Bayesian updating formula for normal distributions
      posterior_var <- 1 / (1/prior_var + 1/obs_var)
      posterior_mean <- posterior_var * (prior_mean/prior_var + obs_mean/obs_var)

      return(list(mean = posterior_mean, var = posterior_var))
}

# Initialize with prior
current_mean <- prior_mean
current_var <- prior_sd^2

```

Appendix 1 *(continued)*

```

# Sequential Bayesian updating with each comparable
for (i in 1:length(comp_values)) {
      update <- bayesian_update(
            prior_mean = current_mean,
            prior_var = current_var,
            obs_mean = comp_values[i],
            obs_var = comp_sds[i]^2
      )

      current_mean <- update$mean
      current_var <- update$var
      current_sd <- sqrt(current_var)

      # Add this step to the tracking dataframe
      calculation_steps <- calculation_steps %>%
            add_row(
                  Step = paste0("After Comp ", LETTERS[i]),
                  Mean = current_mean,
                  SD = current_sd,
                  `Z (95%)` = 1.96,
                  `Z × SD` = 1.96 * current_sd,
                  CI_Lower = current_mean - 1.96 * current_sd,
                  CI_Upper = current_mean + 1.96 * current_sd,
                  `CI Range` = 2 * 1.96 * current_sd
            )
}

# Calculate posterior standard deviation
posterior_sd <- sqrt(current_var)

# Calculate 95% confidence intervals
prior_ci_lower <- prior_mean - 1.96*prior_sd
prior_ci_upper <- prior_mean + 1.96*prior_sd
posterior_ci_lower <- current_mean - 1.96*posterior_sd
posterior_ci_upper <- current_mean + 1.96*posterior_sd

# Round all values to nearest thousand
prior_mean_rounded <- round(prior_mean/1000)*1000
prior_sd_rounded <- round(prior_sd/1000)*1000
prior_ci_lower_rounded <- round(prior_ci_lower/1000)*1000
prior_ci_upper_rounded <- round(prior_ci_upper/1000)*1000

posterior_mean_rounded <- round(current_mean/1000)*1000
posterior_sd_rounded <- round(posterior_sd/1000)*1000
posterior_ci_lower_rounded <- round(posterior_ci_lower/1000)*1000
posterior_ci_upper_rounded <- round(posterior_ci_upper/1000)*1000

# Format confidence intervals as ranges
```

Appendix 1 *(continued)*

```
prior_ci_formatted <- paste(dollar(prior_ci_lower_rounded), "-",
dollar(prior_ci_upper_rounded))
posterior_ci_formatted <- paste(dollar(posterior_ci_lower_rounded), "-",
dollar(posterior_ci_upper_rounded))

# Create summary table
summary_table <- tibble(
      Estimate = c("Prior (Initial)", "Posterior (Updated)"),
      `Mean Value` = c(dollar(prior_mean_rounded),
dollar(posterior_mean_rounded)),
      `Standard Deviation` = c(dollar(prior_sd_rounded),
dollar(posterior_sd_rounded)),
      `95% Confidence Interval` = c(prior_ci_formatted,
posterior_ci_formatted)
)

# Create comparable sales table
comps_table <- tibble(
      Property = c("Subject", "Comparable A", "Comparable B", "Comparable
C"),
      `Sale Price` = c("- (n/a)", dollar(300000), dollar(310000),
dollar(330000)),
      `Living Area (SF)` = c(2000, 2100, 1900, 2000),
      `Year Built` = c(2000, 1995, 2005, 2000),
      `Has Pool?` = c("No", "No", "No", "Yes")
)

# Create adjusted values table
adjusted_table <- tibble(
      `Comparable` = c("Comparable A", "Comparable B", "Comparable C"),
      `Original Price` = c(dollar(300000), dollar(310000), dollar(330000)),
      `Adjustment` = c(dollar(-5000), dollar(5000), dollar(-20000)),
      `Adjusted Value` = c(dollar(295000), dollar(315000), dollar(310000)),
      `Uncertainty (±)` = c(dollar(15000), dollar(10000), dollar(12000))
)

# Format the detailed calculation steps table with rounded values
detailed_calculation_rounded <- calculation_steps %>%
      mutate(
            Mean_rounded = round(Mean/1000)*1000,
            SD_rounded = round(SD/1000)*1000,
            `Z_x_SD_rounded` = round(`Z × SD`/1000)*1000,
            CI_Lower_rounded = round(CI_Lower/1000)*1000,
            CI_Upper_rounded = round(CI_Upper/1000)*1000,
            `CI_Range_rounded` = round(`CI Range`/1000)*1000
      ) %>%
      mutate(
            Mean_display = dollar(Mean_rounded),
            SD_display = dollar(SD_rounded),
```

Appendix 1 *(continued)*

```
        `Z_x_SD_display` = dollar(`Z_x_SD_rounded`),
        `CI Calculation` = paste0("Mean ± Z × SD = ", Mean_display, " ±
", `Z_x_SD_display`),
        `95% Confidence Interval` = paste(dollar(CI_Lower_rounded), "–",
dollar(CI_Upper_rounded)),
        `Range (±)` = paste0("±", dollar(`CI_Range_rounded`/2))
    ) %>%
    select(Step, Mean_display, SD_display, `Z (95%)`, `CI Calculation`,
`95% Confidence Interval`, `Range (±)`) %>%
    rename(
        Mean = Mean_display,
        SD = SD_display
    )

# Format the summary table with GT
summary_gt <- summary_table %>%
    gt() %>%
    tab_header(
        title = "Prior vs. Posterior Value Estimate"
    ) %>%
    cols_align(
        align = "left",
        columns = Estimate
    ) %>%
    cols_align(
        align = "right",
        columns = c(`Mean Value`, `Standard Deviation`, `95% Confidence
Interval`)
    ) %>%
    tab_style(
        style = cell_borders(
            sides = "bottom",
            weight = px(2)
        ),
        locations = cells_column_labels()
    )

# Format the comparables table with GT
comps_gt <- comps_table %>%
    gt() %>%
    tab_header(
        title = "Comparable Sales Data"
    ) %>%
    cols_align(
        align = "left",
        columns = Property
    ) %>%
    cols_align(
        align = "right",
        columns = c(`Sale Price`, `Living Area (SF)`, `Year Built`)
```

Appendix 1 *(continued)*

```
      ) %>%
      cols_align(
            align = "center",
            columns = `Has Pool?`
      ) %>%
      tab_style(
            style = cell_borders(
                  sides = "bottom",
                  weight = px(2)
            ),
            locations = cells_column_labels()
      )

# Format the adjusted values table with GT
adjusted_gt <- adjusted_table %>%
      gt() %>%
      tab_header(
            title = "Comparable Adjustments and Uncertainties"
      ) %>%
      cols_align(
            align = "left",
            columns = Comparable
      ) %>%
      cols_align(
            align = "right",
            columns = c(`Original Price`, `Adjustment`, `Adjusted Value`,
`Uncertainty (±)`)
      ) %>%
      tab_style(
            style = cell_borders(
                  sides = "bottom",
                  weight = px(2)
            ),
            locations = cells_column_labels()
      )

# Format the detailed calculation steps table with GT
detailed_calculation_gt <- detailed_calculation_rounded %>%
      gt() %>%
      tab_header(
            title = "Detailed Confidence Interval Calculations",
            subtitle = "Step-by-step progression of Bayesian updating"
      ) %>%
      cols_align(
            align = "left",
            columns = c(Step, `CI Calculation`)
      ) %>%
      cols_align(
            align = "right",
            columns = c(Mean, SD, `Z (95%)`, `95% Confidence Interval`,
```

Appendix 1 *(continued)*

```
`Range (±)`)
      ) %>%
      tab_style(
            style = cell_borders(
                  sides = "bottom",
                  weight = px(2)
            ),
            locations = cells_column_labels()
      ) %>%
      tab_footnote(
            footnote = "Z-score of 1.96 corresponds to 95% confidence level
in a normal distribution",
            locations = cells_column_labels(columns = `Z (95%)`)
      ) %>%
      tab_footnote(
            footnote = "The confidence interval calculation follows the
formula: Mean ± (Z × Standard Deviation)",
            locations = cells_column_labels(columns = `CI Calculation`)
      ) %>%
      tab_footnote(
            footnote = "Range (±) shows the plus/minus value from the mean,
rounded to nearest $1,000",
            locations = cells_column_labels(columns = `Range (±)`)
      )

# Print the tables
comps_gt
adjusted_gt
detailed_calculation_gt
summary_gt

# Save the tables as HTML files (can be copied into Word)
gtsave(comps_gt, "comparable_sales_table.html")
gtsave(adjusted_gt, "adjustments_table.html")
gtsave(detailed_calculation_gt, "detailed_calculation_table.html")
gtsave(summary_gt, "value_estimate_table.html")
```

Appendix 2. Sample Python Code

```
try:
    import numpy as np
    import pandas as pd
except ModuleNotFoundError as e:
    print(f"Module not found: {e.name}. Please ensure numpy and pandas are installed in your environment.")
    raise

# Set seed for reproducibility
np.random.seed(123)

# Prior distribution parameters
prior_mean = 300000
prior_sd = 30000

# Comparable sales data after adjustments
comp_values = [295000, 315000, 310000]  # Adjusted values
comp_sds = [15000, 10000, 12000]        # Uncertainties for each comp

# Function to perform Bayesian update with one comparable
def bayesian_update(prior_mean, prior_var, obs_mean, obs_var):
    posterior_var = 1 / (1/prior_var + 1/obs_var)
    posterior_mean = posterior_var * (prior_mean/prior_var + obs_mean/obs_var)
    return posterior_mean, posterior_var

# Initialize with prior
current_mean = prior_mean
current_var = prior_sd ** 2

# Store calculation steps
calculation_steps = [["Prior", current_mean, np.sqrt(current_var)]]

# Sequential Bayesian updating with each comparable
for i, (comp_value, comp_sd) in enumerate(zip(comp_values, comp_sds), 1):
    current_mean, current_var = bayesian_update(current_mean, current_var, comp_value, comp_sd**2)
    calculation_steps.append([f"After Comp {chr(64 + i)}", current_mean, np.sqrt(current_var)])

# Convert results to DataFrame
df_results = pd.DataFrame(calculation_steps, columns=["Step", "Mean", "SD"])

# Compute confidence intervals
df_results["CI Lower"] = df_results["Mean"] - 1.96 * df_results["SD"]
df_results["CI Upper"] = df_results["Mean"] + 1.96 * df_results["SD"]
df_results["CI Range"] = 2 * 1.96 * df_results["SD"]

# Round and format output
df_results = df_results.round(-3)

# Display results
print(df_results)
```